Problem Set 7

1. For the Adams Bashforth second order method

$$u_{n+1} = u_n + \frac{1}{2}h(3u'_n - u'_{n-1})$$

   (a) Compute a table of the numerical values of the σ-roots of the 2nd order Adams Bashforth method (AB2) when $\lambda = i$.

   (b) Take h in intervals of 0.05 from 0 to 0.80 and compute the absolute values of the roots to at least 6 places.

   (c) Plot the trace of the roots in the complex σ-plane and draw the upper right hand quadrant of the unit circle on the same plot.

   (d) Repeat the above for the RK2 method.

2. Consider the above two methods. Use a step size of 0.2 and advance 100 time steps. At the end of this interval:

   (a) What is the magnitude of the global error in amplitude?

   (b) What is the magnitude of er_ω?

   (c) On the basis of this information, which is the best method to use for a pure convection problem?

3. Consider the PDE

$$\frac{du}{dt} + a\frac{du}{dx} = \nu\frac{d^2u}{dx^2}$$

   in which $\nu > 0$ and $-\infty \leq a \leq \infty$. Use the 3–pt central difference scheme for the second derivative in space and use the approximation

$$(\delta_x u)_j = \frac{1}{\Delta x}(u_{j+1} - u_j)$$

   for the first derivative approximation in space.

   (a) Write the banded matrix difference operator for the combined difference approximations.

   (b) Find λ_m for the resulting ODE.

   (c) What is the range of a for which the method is inherently stable?

4. The widely known Lax–Wendroff method when appplied to the model equation $\frac{du}{dt} + a\frac{du}{dx} = 0$ gives:

$$u_j^{n+1} = u_j^n - \frac{1}{2}C_n(u_{j+1}^n - u_{j-1}^n) + \frac{1}{2}C_n^2(u_{j+1}^n - 2u_j^n + u_{j-1}^n)$$

   where C_n, known as the CFL number, is $a\Delta t/\Delta x$. Using the Fourier or von Neumann stability analysis, find the range of C_n for which the method is stable.